Fabric Texture Analysis using Wavelet Packet Spectrum and Fourier Spectrum

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Abstract—Two Dimensional spectrum estimator from the statistical properties of wavelet packet coefficients of random process is derived in this paper. This Wavelet Packet Spectrum estimator has been applied to the fabric textures and the performance is compared with conventional 2-D Fourier-based spectrum estimator on same fabric texture for content based image retrieval. The results illustrate the effectiveness of the wavelet-based spectrum estimation.

Keywords— *2-D Wavelet Packet Spectrum (WPS), random fields, spectral analysis, spectrum estimation, Fourier Spectrum (FS), texture.*

I. INTRODUCTION

Fabric textures are the structures of interwoven fibers or surface of other elements. The present paper investigates the capability of 2-D wavelet packets to yield an accurate Power Spectral Density (PSD) estimator for characterizing second order statistical properties of random fields. In this brief, we consider the standard Fourier Spectrum as benchmark and seek for estimating this spectrum by using suitable wavelets.

This positions the Shannon wavelet at the focus of the paper: as the decomposition level tends to infinity, the bias of the Shannon wavelet spectrum estimator tends to 0. The bias of an arbitrary wavelet spectrum estimator then relates to the closeness of the wavelet under consideration to the Shannon wavelet. This closeness is measured through a parameter called wavelet order.

The Wavelet Packet Spectrum is compared with Fourier Spectrum in order to show WPS is better than FS. The paper focuses on presenting the specificities that follow by dimensionality increasing from:

- The analytic form of the wavelet packet based PSD,
- The singular paths of Fractional Brownian fields,
- The 2-D wavelet packet PSD estimator.

Furthermore, for textured image analysis, the paper provides experimental results for evaluating the relevance of spectrum estimation by

- Comparing PSD estimated from Fourier and wavelet packet methods and
- Performing content based image retrieval associated with spectral similarity measurements in the Fourier and wavelet packet domains.

II. FOURIER SPECTRUM FOR TEXTURE ANALYSIS

A spectrum is a relationship typically represented by a plot of the magnitude or relative value of some parameter against frequency. Every physical phenomenon, whether it is an electromagnetic, thermal, mechanical, hydraulic or any other system, has a unique spectrum associated with it. In electronics, the phenomena are dealt with in terms of signals, represented as fixed or varying electrical quantities of voltage, current and power. These quantities are typically described in the time domain and for every function of time, f(t), an equivalent frequency domain function F(ω) can be found that specifically describes the frequency-component content (frequency spectrum) required to generate f(t). A study of relationships between the time domain and its corresponding frequency domain representation is the subject of Fourier analysis and Fourier transforms.

The forward Fourier transform, time to frequency domain, of the function x(t) is defined

$$F[x(t)]=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = X(\omega) \quad (1)$$

and the inverse Fourier transform , frequency to time domain, of X(ω) is

$$F^{-1}[X(\omega)]=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega = x(t) \quad (2)$$

In order to estimate the Fourier Spectrum we have to consider below steps by considering framework

1. Divide the data sequence x(n) into $L \leq N/M$ segments, $x^{l}_{M}(n)$
2. Multiply a segment by an appropriate window
3. Take the Fourier transform of the product
4. Multiply procedure 3 by its conjugate to obtain the spectral density of the segment
5. Repeat procedures 2 through 4 for each segment so that the average of these periodogram estimates produce the Power Spectral Density estimate, equation.

III. WAVELET PACKET SPECTRUM ESTIMATOR

A. Introduction on 2-D Wavelet Packets:

We consider the 2-D separable wavelet packet decomposition in a continuous time signal setting for presenting theoretical results [6]. Advanced concepts and algorithms concerning 1D and 2-D wavelet packet analysis can be found in [6]. The reader is also invited to refer to [7], [8] (wavelets) and [4], [9] (wavelet packets) for more details on the statistical properties of wavelet transforms, when the decomposition relates to a random process. In this decomposition, the wavelet paraunitary filters H_0 (low-pass, scaling filter) and H_1 (high-pass, wavelet filter) are used to split the input functional space $\mathbf{U} = \mathbf{W}_{0,0} \subset L^2(R^2)$ into orthogonal subspaces.

Assume that the scaling filter is with order r: $H_0 \equiv Hr^0$, where r is the largest non-negative integer

$$Hr^0(\omega) = \left(\frac{1+e^{-i\omega}}{2}\right)^r Q(e^{i\omega}) \qquad (3)$$

filter H_0^s denoting the scaling filter associated with the Shannon wavelet. Then the 1D multiscale filters (H^r j,ni)i=1,2 have very tight supports when r is large.

B. Wavelet Packet Paths

This section presents a specific wavelet packet path description derived from the binary sequence approach of [11] for representing nested wavelet packet subspaces. This description is suitable for establishing asymptotic properties of 2-D wavelet packets with respect to the increase of the decomposition level. It is worth mentioning that some specific paths will present singular behavior, depending on the input random field: The wavelet coefficients of certain non-stationary random fields on the sub bands associated with these singular paths will remain non-stationary. As a matter of example,

1) The separable Fractional Brownian field analyzed which admits frequency indices n(j) such that n_1(j) = 0 (resp. n_2(j) = 0) for every j as singular frequency indices. The set of (singular) paths associated with these frequency indices will be denoted by $Pn/n_1=0$ (resp. $Pn/n_2=0$).
2) The isotropic Fractional Brownian field analyzed which admits a unique singular path: the approximation path denoted by P_0 and associated with frequency indices nP_0 (j) = 0 for every j .

C. 2-D Wavelet Packet based Spectrum Estimation

The Wavelet Packet Spectrum is estimated by using Shannon wavelet. The Shannon wavelet is defined by $\Phi(t)=\mathrm{sinc}(t/2)\cos(3\pi t/2)$ and its Fourier transform

$$\hat{\varphi}(f) = \begin{cases} 1 & 0.5 \le |f| \le 1 \\ 0 & otherwise \end{cases} \qquad (4)$$

The properties of Shannon wavelet are

1. It can be used for both complex and simple signals
2. It gives accurate results than other wavelets.

The Shannon wavelet is as shown in Figure 1

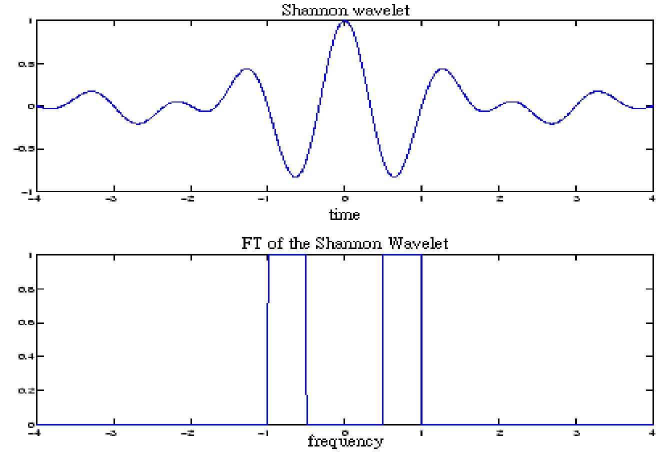

Fig.1: Shannon wavelet

The spectrum estimation method presented in this section follows from the asymptotic analysis of the autocorrelation functions of the 2-D wavelet packet coefficients. This asymptotic analysis is performed with respect to the wavelet order r and the wavelet decomposition level j. When r increases, the asymptotic behavior of the sequence of wavelet functions is driven by the Shannon wavelet functions. In this respect, we consider the Shannon wavelets in below and derive asymptotic results with respect to the wavelet decomposition level. The case of a wavelet with order r will be considered as an approximation of the Shannon limiting behavior when r is large enough.

D. 2-D Wavelet Packet based Spectrum Estimation

The following provides a non-parametric method for estimating spectrum γ of 2-D random fields on the basis of the convergence criteria. It follows that

$\gamma(\omega_1[P],\omega_2[P]) = \lim_{j\to+\infty} R^S{}_{j,n}[0, 0]$ so that the continuity points of spectrum γ can be estimated by sub band variances (values{ $R^S{}_{j,n}[0, 0]$} provided that the Shannon wavelet is used and j is large enough. Furthermore, we can derive from the convergence criteria, several spectrum estimators by considering wavelets with finite orders r (Shannon wavelet corresponds to r $= +\infty \equiv$ S), the accuracy of the spectrum estimation being dependent on the wavelet order as shown in Proposition 3 below. Assuming a uniform sampling (regularly spaced frequency plane tiling), the method applies upon the following steps.

1) Define a frequency grid compose with frequency points $(\frac{P_1\pi}{2i}, \frac{P_2\pi}{2^i})$ for $P_1,P_2 \in \{0, 1, \ldots, 2^j - 1\}$ (natural ordering).
2) Compute, the index $n \in \{0, 1, \ldots, 4^j - 1\}$ (Corresponding to the wavelet packet ordering) associated with (P_1, P_2).
3) Set, for any pair (P_1, P_2) given in step 1) and the corresponding n obtained from step 2).

There are three propositions to find Wavelet based Spectrum as shown below:

Proposition 1: The height of this fractional moment depends on the scaling function associated with the wavelet packet decomposition. Note also that when both n1 = n2 = 0, the non-stationary in wavelet coefficients is more intricate, mainly because the analyzing function has no vanishing moments.

Proposition 2: The autocorrelation function of the wavelet packet coefficients of separable and isotropic Fractional Brownian Field can be written in the integral form.

Proposition 3: We derive that the bias of the estimator given by depends on the decomposition level and wavelet order used. This bias tends to 0 when both j and r tends to infinity.

IV. Application of Wavelet Packet Spectrum to Fabric Texture

The below shown Figure 2 is one of the Fabric texture and then

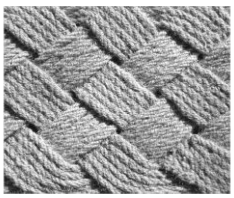

Fig.2: Fabric texture

The Wavelet Packet Spectrum estimator is applied to this texture information to derive Wavelet Packet Spectrum of this texture which is as shown below Figure 3:

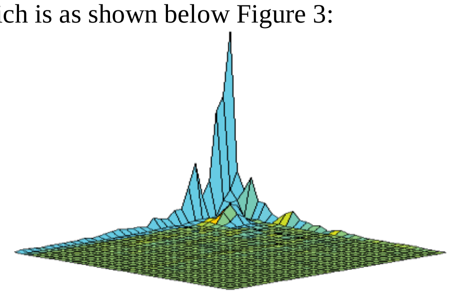

Fig.3: Wavelet Packet Spectrum for above fabric texture

The comparison should be made with Fourier Spectrum which is shown in Figure 4.

FT-PSD Fabric Texture WPT-PSD

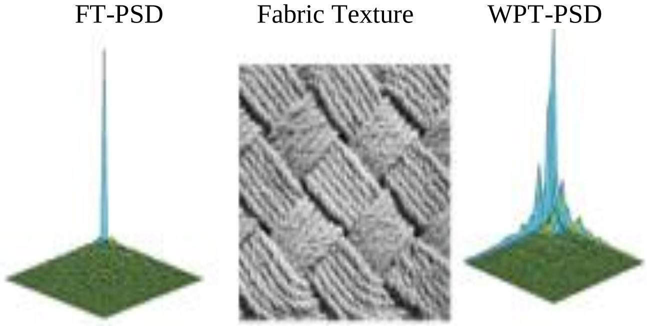

Fig4: Textures image and their spectra γ computed by using discrete Fourier and wavelet packet transforms. Abscissa of the spectra images consist of a regular grid over $[0, \pi/2] \times [0, \pi/2]$.

Note: Colors represented in Figure 4 are simulated from a light source in order to ease 3-D visualization: red color [value 1] corresponds to fully illuminated shapes whereas blue color [value 0] is associated to shaded areas, green color corresponds to value 0.5.

V. Advantages of Proposed Method

1. The over complete structure of WPT provides flexibility for the signal representation to achieve better classification accuracy.
2. The best basis provides the most suitable frequency sub bands for the signal representation.
3. The subject-based adaptation feature extraction with this method constructs a wavelet packet best basis fitted for each object and so it can find the suitable and specific features for a subject's signals.

VI. Applications of Wavelet Packet Spectrum

1. Hurst parameter estimation for self-similar medical images, see for instance [2].
2. Texture modeling by using Wold decompositions estimation the poles of the spectrum is necessary to determine the spectral singularities involved in the deterministic texture contribution. These poles are associated with peaks of the spectrum and their number, as well as their location determines the accuracy of the modeling.
3. Spread-spectrum image watermarking.

VII. Experimental Results with Conclusion

Two issues are addresses in the paper: (i) estimating the PSD from the statistical properties of the wavelet packet coefficients, (ii) applying to texture, (iii) comparing the results with Fourier Spectrum. Issue (i) has been tackled from asymptotic properties of the Shannon wavelet packets and the spectrum estimation method proposed is more effective for wavelet filters with large order. This section provides experimental results on spectral analysis of textures. A Wavelet Packet Spectrum of fabric texture image is provided in Figure.3. The Wavelet Packet Spectra have been computed with the decomposition level is 6 and the Daubechies wavelet with order r = 7 is used. Spectra computed from the Fourier transform are also given in this figure 4, for comparison purpose. From a visual analysis of images, one can remark that most of this texture exhibit non overlapping textons replicating repeatedly: thus, coarsely, we can distinguish several frequencies having significant variance contributions (from a theoretical consideration), when the texture does not reduce to the replications of a single texton. In addition, when these textons occupy approximately the same spatial area (see for instance “Fabric” textures in Figure 2), the frequencies with high variance contributions (peak in the spectrum) are close in terms of their spatial location (from a theoretical consideration).

The above heuristics, issued from visual image analysis, are confirmed by considering the wavelet packet spectra (see for instance spectra of "Fabric" textures in Figure 2), whereas, in most cases, the two dimensional discrete Fourier transform exhibits only one peak. One can highlight that the poorness of the Fourier spectra is not due to a lack of resolution in the sampling step of the Fourier transform. This poorness can be explained by noting that Fourier transform is sensitive to global spatial regularity. In contrast wavelet packets can capture local spatial regularity and lead to a more informative spectrum estimator when several frequencies contribute in texture variance distribution.

VIII. References

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